Quiz 3
Coding Theory

10^{th} February 2006

Time: 1 hour (12:30–1:30pm)

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 3 & 1 \\ 2 & 4 & 4 \end{bmatrix}$$

over $GF(5)$. Show that A can give minimum distance separable (MDS) codes. [6] Find two such codes if they exist, and give either a generator matrix or a parity check matrix for each of them. [6] Then give the code words and encoding functions for each. [8]

Solution. Examine the values of determinant for all submatrices of A, $\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 2$, $\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} = 3$, $\begin{vmatrix} 3 & 4 \\ 3 & 1 \end{vmatrix} = 1$, $\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 3$, $\begin{vmatrix} 1 & 4 \\ 2 & 4 \end{vmatrix} = 1$, $\begin{vmatrix} 3 & 4 \\ 4 & 4 \end{vmatrix} = 1$, $\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 2$, $\begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} = 1$, $\begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix} = 3$, $\begin{vmatrix} 1 & 3 & 4 \\ 2 & 3 & 1 \\ 2 & 4 & 4 \end{vmatrix} = 3$. We can see that no submatrices of A are singular, therefore we may obtain from A two MDS codes, namely the $[6, 3, -]$ code over $GF(5)$ with the generator matrix $G = (I_3 \quad A)$ and the $[6, 3, -]$ code over $GF(5)$ with the parity check matrix $H - (A \quad I_3)$. For the first one, the generating function is

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 3 & 4 \\ 0 & 1 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 4 & 4 \end{pmatrix}$$

Then,

$$(\, a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \,) = (\, a_1 \quad a_2 \quad a_3 \,) \begin{pmatrix} 1 & 0 & 0 & 1 & 3 & 4 \\ 0 & 1 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 4 & 4 \end{pmatrix}$$

and the encoding functions becomes

$$\begin{aligned} a_4 &= a_1 + 2a_2 + 2a_3 \\ a_5 &= 3a_1 + 3a_2 + 4a_3 \\ a_6 &= 4a_1 + a_2 + 4a_3 \end{aligned}$$

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The code words are $C = \{100134; 010231; 001244; 110310; 101323; 11420\}$.

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For the $[6, 3, -]$ code, from the parity check matrix we have the generating function

$$G = (\, I_3 \quad -A^T \,) = \begin{pmatrix} 1 & 0 & 0 & 4 & 3 & 3 \\ 0 & 1 & 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 4 & 1 \end{pmatrix}$$

Then,

$$(\, a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \,) = (\, a_1 \quad a_2 \quad a_3 \,) \begin{pmatrix} 1 & 0 & 0 & 4 & 3 & 3 \\ 0 & 1 & 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 4 & 1 \end{pmatrix}$$

and the encoding functions become

$$\begin{aligned} a_4 &= 4a_1 + 2a_2 + a_3 \\ a_5 &= 3a_1 + 2a_2 + 4a_3 \\ a_6 &= 3a_1 + a_2 + a_3 \end{aligned}$$

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And then the code is $C = \{100433, 010221, 001141, 110104, 101024, 011312\}$.

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